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## Monterey, California



A FIGURE OF MERIT FOR NUWES DATA

by

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>A measure of quality for both raw and smoothed 3-D data collected at NUWES is proposed and illustrated in this report. This measure, called a Figure-of-Merit is based on polynomial fitting by the Least-Squares method to data segments. It is derived by establishing confidence intervals for the differences between observed and smoothed (estimated) values of vehicle coordinates at the observation times. The proposed Figure-of-Merit provides numerical (statistical) bounds for this difference. |  |                                                                                 |                               |



A FIGURE OF MERIT

FOR

NUWES DATA

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## ABSTRACT

A measure of quality for both raw and smoothed 3-D data collected at NUWES is proposed and illustrated in this report. This measure, called a Figure-of-Merit, is based on polynomial fitting by the Least-Squares method to data segments. It is derived by establishing confidence intervals for the differences between observed and smoothed (estimated) values of vehicle coordinates at the observation times. The proposed Figure-of-Merit provides numerical (statistical) bounds for this difference.

Keywords: Measure of quality, Figure-of-Merit, polynomial fit, statistical bounds.

## I INTRODUCTION

There is a need for Proof and Test to provide users of NUWES data with estimates of actual vehicle positions as a function of time and also to provide some indication of the quality of those estimates (that is, to provide some indication of how close those estimates are to the actual positions). There is also a need to provide Instrumentation with feedback on the capability of the position location system for providing data satisfying the needs of the users. The purpose of this report is to propose a measure of quality which will be called a Figure of Merit to satisfy those needs.

The proposed Figure of Merit (FM) is based on the statistical concept of a confidence interval. It is a numerical value for a statistical bound on the difference between the estimated coordinate value ( $x_e$ ) and the actual or true value ( $x_t$ ). In lay terms, the values of FM provide readily understandable indicators of the quality of the estimates and hence of the data used to establish them.

Application of the proposed FM is illustrated for a 40 point segment of NUWES data recorded on a trial run involving two vehicles. The data segment presents a fairly wide range of difficulty for both the position location system and the smoothing process.

## II THE PROPOSED FIGURE OF MERIT

The proposed FM incorporates the occurrence of missing and outlier points as well as the magnitude of scatter or noise in the data segment to establish a measure of the quality of the data segment used to produce the estimate  $x_e$  at the observation time in the center of the segment. A 7-point Least-Squares Polynomial model (Ref. 4) is used in the smoothing process to determine values for the estimated values (the  $x_e$ 's) and will also provide a basis for establishing values for the FM's to be associated with the  $x_e$ 's. Each of the factors contributing to the FM's is described below.

1. Missing data points in a data segment degrade the quality of the estimates by decreasing the number of legitimate observations in the 7-point segments used to establish the estimates. The smoothing process requires that temporary values be provided at the missing points so that a sequence of seven consecutive observations are available for the smoothing. These temporary values are produced by linear averaging of the adjacent legitimate observations. Smoothing of these temporary values is repeated until the residual error (the difference between the presmoothed values and the smoothed values at the smoothing point) is within acceptable bounds. This bound has been set at unity (1) so that the residual error is well within the noise level present in good quality data and hence does not contaminate the information provided by the legitimate observations in the data segment to a serious extent.

2. Outlier data points are observations that are inconsistent with neighboring values. These are identified by using sequential differences (Ref.2) with any observation having a fourth order difference of 50 in magnitude being identified as a potential outlier.

Since outliers also contaminate the fourth order differences of adjacent observations but to a lesser extent, the largest fourth order difference exceeding the threshold is identified as the only outlier. After smoothing this outlier, sequential differences should be recalculated to determine whether other outliers occur in a data segment. As with missing points, the smoothing process should be iterated (repeated) until the residual error at that point is negligible.

The number of legitimate observations in a 7-point data segment to be used for estimating the value  $x_e$  is

$$NS = 7 - M - W$$

where  $M$  is the number of missing points and  $W$  is the number of outliers or wild values in the segment.

3. A measure of the scatter in a data segment is obtained in the smoothing process. This is the standard deviation SDRK of the residual errors of the data segment when  $K$ th order polynomial is fitted to the segment. SDRK is determined by finding the sum of the squares of the residual errors (SSRK) and the appropriate degrees of freedom DFK where

$$\begin{aligned} NS - 2 & \text{ for } k = 1, \\ DFK = NS - 3 & \text{ for } k = 2, \\ NS - 4 & \text{ for } k = 3. \end{aligned}$$

(Note that 2,3,4 are the number of parameters in polynomials of order 1,2,3 and are called the degrees of freedom lost when polynomials of those orders are fitted to the data segment.) SDRK is defined as

$$SDRK = \text{SQR} (SSRK/DFK).$$

It can be established that

$$SSR1 \geq SSR2 \geq SSR3 \geq \dots$$

On the other hand,

$$DF1 > DF2 > DF3 > \dots$$

Thus it is possible, for example, for a second order polynomial ( $k=2$ ) to have a smaller standard deviation than a third order polynomial ( $k=3$ ). In previous work on this project selection of the appropriate order polynomial for fitting a data segment was made by comparison of the values of the SDRK's. Note that the SSRK's represent the quality of mathematical fit of the polynomials to the data segment whereas the SDRK's represent the quality of statistical fit. Subsequently, it will be shown that the values of the FM's (yet to be defined) provide an even better basis for selection of the polynomial order to be used to fit a data segment.

Attention should be directed to the role of the DFK's in establishing the SDRK's. If DFK is less than unity ( $DFK < 1$ ) for any  $K$ , then a polynomial of order  $K$  cannot be fitted to the data segment using the L-S Method. For example, if there are 3 missing and outlier points in a 7-point data segment, then  $NS = 4$  and a polynomial of order  $K=3$  would have  $DF3 = 0$  as its degrees of freedom. (A polynomial of order three can, at least theoretically, be fitted exactly to the four legitimate observations in the segment but the noise in those observations is also included in the fitting and no estimate of the magnitude of the noise can be made.) One other point should be stressed here. Suppose that a cubic polynomial is appropriate for the vehicular path but that  $DF3 < 1$ . The value of SDR1 and SDR2 include not only the effects of scatter but also components due to the inappropriateness of the model. Instrumentation should be aware of the fact that the SDR's do not represent only scatter but can contain model error components.

A final factor needs discussion before confidence intervals and FM's can be presented. Establishment of a confidence interval requires knowledge of a specific statistical distribution. In this application, appropriate assumptions

lead to the Student T distribution for the residual errors. An extract of a table for this distribution (Ref. 7) is presented below when a confidence level of 0.95 is selected.

|        |   |        |       |       |       |       |       |
|--------|---|--------|-------|-------|-------|-------|-------|
| DFK    | : | 0      | 1     | 2     | 3     | 4     | 5     |
| T(DFK) | : | 99.999 | 6.314 | 2.920 | 2.353 | 2.132 | 2.015 |

The value of T (0) should be infinity to indicate no confidence in the estimate. The value 99.999 has been introduced somewhat arbitrarily for computational convenience and will result in a wide confidence interval and a large value for FM.

At last the background is set for the introduction of confidence intervals and the definition of the proposed Figure of Merit FM. A confidence interval for the true positional coordinate at any time T is specified by its two end-points, i.e.,

$$CI(x_t) = (x_e - T(DFK) * SDRK / SQR(NS), x_e + T(DFK) * SDRK / SQR(NS))$$

Again in lay terms, this expression indicates that the actual value of position coordinate can be expected to be in this interval centered at the estimated value  $x_e$  about 95% of the time.

The proposed measure of quality is the statistical bound

$$FMK = T(DFK) * SDRK / SQR(NS)$$

for the difference between the actual value and the estimated value of a position coordinate. This difference can be expected to be less than FM about 95% of the times that such bounds are calculated. Large values of FMK indicate

low confidence that  $x_e$  is close to  $x_t$  and small values of  $FM_K$  indicate that  $x_t$  should differ only slightly from  $x_e$ .

It would appear reasonable to shift the basis for selecting the appropriate order of the polynomial to be used to fit a data segment to that  $K$  which produces the smallest bound for the difference between  $x_e$  and  $x_t$ . Thus, the proposed figure of merit is

$$FM = \min(FM_K).$$

$K$

Also, for this specific  $K$ ,

$$DF = DF_K,$$

and

$$SDR = SDR_K.$$

### III APPLICATION

Calculation of FM's will be illustrated for a specific sample of NUWES data (investigator's sample 2.1AX).

The sample data selected includes 40 observation times ( $t=2121$  to  $t=2160$ ). In order to smooth and establish an estimate  $x_e$  and an FM at each of these times, additional points were needed. Data for times  $t=2117$  to  $t=2163$  is presented in the first three columns of Table 1 and plotted in Figure 1. The first column contains the observation times,  $T$ . The second column gives the identity of the position location array providing the observation with -2 denoting that no observation is available. The third column contains the observed values (the  $x_o$ 's). Note that there are 6 times with missing observations that have been filled by temporary values as mentioned in Section II and to be discussed later.

In order to make the procedure clear, each step is described in some detail below.

STEP 1 Establish temporary values for the  $x$  coordinate ( $x_o$ ) at the missing times. This was accomplished by using linear interpolation between adjacent observed values. thus, for example, the temporary value at time  $t=2120$  was supplied by taking the average of the  $x_o$  values at times  $t=2119$  and  $t=2121$ . Missing points are identified as Questionable values (QP) in column 4 of Table 1. Values of -1 and -2 are used to indicate Unscheduled and Scheduled missing points, respectively. They are also identified by circles in Figure 1.

FIGURE 1  
NWS 2A1X

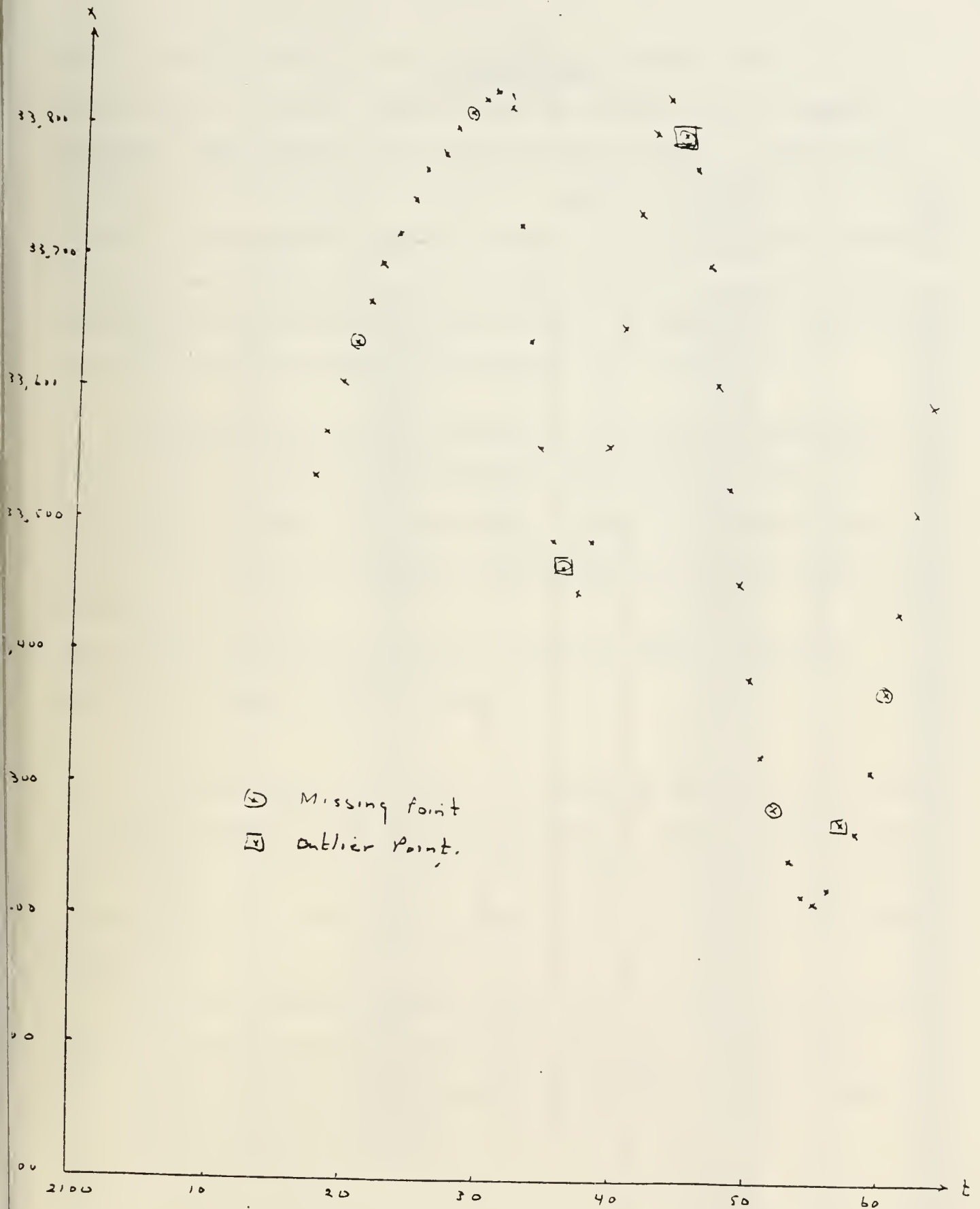


TABLE 1  
Sample NWS2A1X

| T    | ARRAY | xo        | QP  | K | DF | xe      | R      | FM    |
|------|-------|-----------|-----|---|----|---------|--------|-------|
| 2117 | 3     | 33533.2   | 0   |   |    |         |        |       |
| 2118 | 3     | 33567.7   | 0   |   |    |         |        |       |
| 2119 | 3     | 33603.6   | 0   |   |    |         |        |       |
| 2120 | -2    | (33634.1) | -2  | 2 | 3  | 33634.9 | (-0.8) | 1.51  |
| 2121 | 3     | 33664.6   | 0   | 2 | 3  | 33665.3 | -0.7   | 1.58  |
| 2122 | 3     | 33695.0   | 0   | 2 | 3  | 33693.5 | 1.5    | 1.47  |
| 2123 | 3     | 33718.5   | 0   | 3 | 2  | 33720.9 | -2.4   | 2.20  |
| 2124 | 3     | 33745.8   | 0   | 2 | 4  | 33744.6 | 1.2    | 2.02  |
| 2125 | 3     | 33767.1   | 0   | 2 | 3  | 33764.6 | 2.5    | 3.14  |
| 2126 | 3     | 33780.7   | 0   | 2 | 3  | 33783.7 | -3.0   | 3.26  |
| 2127 | 3     | 33798.0   | 0   | 3 | 2  | 33799.7 | -1.7   | 3.13  |
| 2128 | -2    | (33810.9) | -2  | 3 | 2  | 33816.3 | (5.4)  | 4.34  |
| 2129 | 3     | 33823.7   | 0   | 3 | 2  | 33828.1 | -4.7   | 4.01  |
| 2130 | 3     | 33827.3   | 0   | 3 | 2  | 33821.6 | 5.7    | 7.02  |
| 2131 | 3     | 33794.2   | 0   | 2 | 3  | 33786.6 | 7.6    | 11.69 |
| 2132 | 3     | 33726.1   | 0   | 3 | 3  | 33724.0 | 2.1    | 2.84  |
| 2133 | 3     | 33637.7   | 0   | 3 | 2  | 33640.1 | -2.4   | 2.20  |
| 2134 | 12    | 33556.5   | 0   | 3 | 2  | 33556.9 | -0.4   | 4.27  |
| 2135 | 12    | 33486.6   | 0   | 3 | 2  | 33489.9 | -3.3   | 2.88  |
| 2136 | -2    | (33466.5) | -12 | 3 | 2  | 33452.2 | (14.3) | 3.94  |
| 2137 | 3     | 33446.5   | 0   | 3 | 2  | 33451.1 | -4.6   | 5.45  |
| 2138 | 3     | 33485.1   | 0   | 3 | 2  | 33489.8 | -4.7   | 7.40  |
| 2139 | 3     | 33559.3   | 0   | 3 | 2  | 33560.9 | -1.6   | 2.82  |
| 2140 | 3     | 33650.2   | 0   | 3 | 3  | 33649.8 | 0.4    | 1.95  |
| 2141 | 3     | 33738.5   | 0   | 3 | 2  | 33734.5 | 4.0    | 4.63  |
| 2142 | 3     | 33799.0   | 0   | 3 | 2  | 33795.6 | 3.4    | 6.25  |
| 2143 | 3     | 33825.8   | 0   | 2 | 3  | 33823.1 | 2.7    | 2.33  |
| 2144 | -2    | (33798.7) | -12 | 3 | 2  | 33813.0 | (14.3) | 2.41  |
| 2145 | 3     | 33771.6   | 0   | 3 | 2  | 33767.4 | 4.2    | 5.58  |
| 2146 | 3     | 33698.3   | 0   | 3 | 2  | 33695.6 | 3.7    | 4.33  |
| 2147 | 3     | 33607.7   | 0   | 3 | 2  | 33614.7 | -7.0   | 7.84  |
| 2148 | 12    | 33528.5   | 0   | 3 | 3  | 33529.8 | -1.3   | 4.04  |
| 2149 | 12    | 33455.2   | 0   | 3 | 3  | 33451.7 | 3.5    | 3.67  |
| 2150 | 3     | 33381.2   | 0   | 3 | 2  | 33383.4 | -2.2   | 2.33  |
| 2151 | 3     | 33323.5   | 0   | 2 | 3  | 33325.2 | -1.7   | 3.42  |
| 2152 | -2    | (33285.1) | -2  | 2 | 3  | 33277/9 | (7.2)  | 2.56  |
| 2153 | 3     | 33246.6   | 0   | 3 | 2  | 33243.5 | 3.1    | 2.75  |
| 2154 | 3     | 33219.5   | 0   | 2 | 2  | 33222.1 | -2.6   | 3.58  |
| 2155 | 3     | 33212.5   | 0   | 2 | 2  | 33214.0 | -1.5   | 3.46  |
| 2156 | 3     | 33221.0   | 0   | 3 | 2  | 33218.6 | 2.4    | 2.71  |
| 2157 | 3     | 33273.5   | -10 | 2 | 2  | 33237.8 | 35.7   | 2.37  |
| 2158 | 3     | 33267.7   | 0   | 2 | 2  | 33269.4 | -1.7   | 1.96  |
| 2159 | 3     | 33313.5   | 0   | 3 | 1  | 33312.9 | 0.6    | 1.29  |
| 2160 | -2    | (33374.2) | -2  | 3 | 1  | 33369.0 | (5.2)  | 1.28  |
| 2161 | 3     | 33434.8   | 0   |   |    |         |        |       |
| 2161 | 3     | 33434.8   | 0   |   |    |         |        |       |
| 2162 | 3     | 33510.5   | 0   |   |    |         |        |       |
| 2163 | 3     | 33592.5   | 0   |   |    |         |        |       |

STEP 2 Check for potential outliers. Sequential differences (Ref.2) were calculated for the sample. (these calculations are omitted here.) Values of the fourth order differences (D4) which were greater than 50 in magnitude were considered to indicate potential outliers. When adjacent values of D4 also exceed 50, the observation with the largest D4 is considered to be an outlier. These are labeled by the value -10 in column 4 of Table 1. Temporary values that are also outliers are identified in column 4 by changing the values to -11 and -12 for unscheduled and scheduled missing values, respectively.

The procedure for identifying outliers is illustrated by examining the values of D4 at times  $t=2135$  where  $D4=-88.2$ , at  $t=2136$  where  $D4=108.7$ , and at  $t=2137$  where  $D4=-92.5$ . The observation at  $t=2136$  is considered to be an outlier. Since it is at a scheduled missing point the appropriate value in column 4 is -12

Outliers were also located at located at  $t=2144$  and  $t=2157$ . These three outliers are indicated by boxes in Figure 1.

STEP 3 Treatment of Outliers. Outliers are treated by iterations of the 7-point L-S smoothing method so that the resulting estimate is not contaminated by the outlier value. Iterations were continued until the residual errors (R) at the times of the outlier and any missing points in the data segment were less than unity.

(The results presented here were calculated on a TI-59 calculator and the polynomial order was selected on the basis of the SDRK's. Results may be different when the FM's are used to select the appropriate polynomial order.)

TABLE 2

Sequential Differences  $t = 2136$ 

## A. Before Smoothing

| t    | xo      | D1    | D2   | D3    | D4    |
|------|---------|-------|------|-------|-------|
| 2133 | 33637.7 |       |      |       |       |
| -    | -       | -81.2 | -    |       |       |
| 2134 | 33556.5 | -     | 11.3 | -     |       |
| -    | -       | -69.9 | -    | 38.5  | -     |
| 2135 | 33486.6 | -     | 49.8 | -     | -88.4 |
| -    | -       | -20.1 | -    | -49.9 | -     |
| 2136 | 33466.5 | -     | -.1  | -     | 109.0 |
| -    | -       | -20.2 | -    | 59.1  | -     |
| 2137 | 33446.3 | -     | 59.0 | -     | -82.7 |
| -    | -       | 38.8  | -    | -23.6 | -     |
| 2138 | 33485.1 | -     | 35.4 | -     |       |
| -    | -       | 74.2  | -    |       |       |
| 2139 | 33559.3 | -     |      |       |       |

## B. After Smoothing

| t    | xo      | D1    | D2   | D3   | D4    |
|------|---------|-------|------|------|-------|
| 2133 | 33637.7 |       |      |      |       |
| -    | -       | -81.2 | -    |      |       |
| 2134 | 33556.5 | -     | 11.3 | -    |       |
| -    | -       | -69.9 | -    | 24.2 | -     |
| 2135 | 33486.6 | -     | 35.5 | -    | -17.2 |
| -    | -       | -34.4 | -    | 7.0  | -     |
| 2136 | 33452.2 | -     | 28.5 | -    | 7.2   |
| -    | -       | 5.9   | -    | 14.2 | -     |
| 2137 | 33446.3 | -     | 44.7 | -    | -23.5 |
| -    | -       | 38.8  | -    | -9.3 | -     |
| 2138 | 33485.1 | -     | 35.4 | -    |       |
| -    | -       | 74.2  | -    |      |       |
| 2139 | 33559.3 | -     |      |      |       |

TABLE 3

Sequential Differences  $t = 2157$ 

## A. Before Smoothing

| t    | xo      | D1   | D2    | D3     | D4     |
|------|---------|------|-------|--------|--------|
| 2154 | 33219.5 | -    | -     | -      | -      |
| -    | -       | -7.0 | -     | -      | -      |
| 2155 | 33212.5 | -    | 15.5  | -      | -      |
| -    | -       | 8.5  | -     | 28.5   | -      |
| 2156 | 33221.0 | -    | 44.00 | -      | -130.8 |
| -    | -       | 52.5 | -     | -102.3 | -      |
| 2157 | 33273.5 | -    | -58.3 | -      | 212.2  |
| -    | -       | -5.8 | -     | 109.9  | -      |
| 2158 | 33267.7 | -    | 51.6  | -      | -146.7 |
| -    | -       | 45.8 | -     | -36.8  | -      |
| 2159 | 33323.5 | -    | 14.8  | -      | -      |
| -    | -       | 60.7 | -     | -      | -      |
| 2160 | 33374.2 | -    | -     | -      | -      |

## B. After Smoothing

| t    | xo      | D1   | D2   | D3   | D4    |
|------|---------|------|------|------|-------|
| 2154 | 33219.5 | -    | -    | -    | -     |
| -    | -       | -7.0 | -    | -    | -     |
| 2155 | 33212.5 | -    | 15.5 | -    | -     |
| -    | -       | 8.5  | -    | -6.4 | -     |
| 2156 | 33221.0 | -    | 9.1  | -    | 8.4   |
| -    | -       | 17.6 | -    | 2.4  | -     |
| 2157 | 33238.6 | -    | 11.5 | -    | 2.8   |
| -    | -       | 29.1 | -    | 5.2  | -     |
| 2158 | 33267.7 | -    | 16.7 | -    | -13.1 |
| -    | -       | 45.8 | -    | -7.9 | -     |
| 2159 | 33313.5 | -    | 8.8  | -    | -     |
| -    | -       | 54.6 | -    | -    | -     |
| 2160 | 33368.1 | -    | -    | -    | -     |

The outlier at  $t=2136$  required 4 iterations. In each iteration a cubic polynomial was used so that  $DF=DF=2$ . These values are shown in columns 5 and 6 of Table 1. The value  $FM=FM3=3.94$  for this point indicates a fair amount of uncertainty. (Values of less than 2 occur when the position location arrays are performing well and the polynomial is a good representation of the actual vehicular path.) Nevertheless, the value of FM indicates that the estimate  $x_e$  can be expected to be within 4 units of the true value of  $x_t$  about 95% of the time. The value of FM is given in column 8 of Table 1.

It is of some interest to note that the residual error (R) between the original temporary value  $x_0=33466.5$  and the estimate  $x_e=33452.2$  is  $R=14.3$  which is more than three times as large as the value of FM and hence the indication of an outlier at this time was correct. The value of the residual error is given in column 7.

Calculations of sequential differences before and after smoothing are presented in Table 2. The potential outliers at  $t=2135$ , 2136, and 2137 are no longer present when the temporary value at  $t=2136$  is replaced by its smoothed value.

Treatment of the outlier at  $t=2144$  was similar. Again, four iterations were required. The value of FM (2.41) is quite low and the residual error  $R=14.3$  indicates that the temporary value at this time was inconsistent with its neighboring values. Sequential differences were not recalculated.

The outlier at  $t=2157$  provides a slight variation since the end point of the segment ( $t=2160$ ) is also a missing point. If, in the smoothing process for treating the outlier at  $t=2157$ , this temporary value were treated as an observed value, the L-S smoothing program would give a weight for this temporary value equal to that of the legitimate observed value. In effect, this would give a

weight of 1.5 for the observation at  $t=2159$  and a weight of 0.5 for the observation at  $t=2161$ . In order to avoid this unbalanced weighting, the temporary value at  $t=2160$  was replaced by its smoothed value at each iteration. The smoothing was then continued until the residual errors at both times  $t=2157$  and  $t=2160$  were less than unity. The purpose of this is to insure that only the  $NS=5$  legitimate observations are retained in the smoothing process. The value of FM (2.37) is again quite low. The error  $R=35.7$  supports the decision to consider the observation at  $t=2157$  an outlier. This is also supported by the change in the sequential differences shown in Table 3. Note that for this point  $NS=7-2=5$  and that with  $K=2$  the value of DF is 2.

STEP 4 Treatment of missing points. the treatment of missing points is the same as that for outliers. As with outliers, iterations were continued until the residual errors were less than unity. The results are shown in Table 4 and Table 1.

STEP 5 Treatment of remaining points. The other points in the sample were smoothed without iterations since they were considered to be legitimate observations. In each treatment the smoothed values of the missing and outlier points were used. The values used for NS and the DF's were reduced to represent the number of legitimate points in each data segment. At several times both the residuals and the FM's were quite large (e.g., at  $t=2131$ ,  $R=7.6$  and  $FM=11.7$ ). Reference to Figure 1 suggests that the vehicle was changing course quite rapidly and that these large values may be due to inadequacy of the model (polynomial) rather than to increased scatter. In either case there is a degradation of quality at this point and this degradation is reflected in the large value for FM.

#### IV CONCLUSIONS AND RECOMMENDATIONS

The proposed Figure-of-Merit provides a numerical expression for the quality of NUWES data. It includes the effects of missing points and outliers on the smoothing process. It also includes the effects of the use of an inadequate ordered polynomial in the smoothing process. In essence, the proposed FM provides a numerical bound for the differences that can be expected between the estimated values and the actual values of the vehicles coordinates at the wequence of observational times.

It is suggested that the FM's can be used to represent the quality of data for specific NUWSES trials and for segments of those trials. It can also be used to indicate the capabilities of specific position location arrays. This latter application could include evaluation of data collected by specific arrays over several trials and thus could be used as an indicator of degradation of array capabilities. Further, on sorting of FM's by distance of vehicles from an array, they could be used to establish relationships between these factors.

As previously indicated, the results presented in Table 1 were obtained using a TI-59 calculator with severely limited program capabilities and hence involving considerable operator interaction. The translation and extension of the process to BASIC language for use on an IBM PC has been initiated. It is recommended that this effort be continued.

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